



Research Seminar Program UC|UP Joint PhD Program in Mathematics

Date: November 30th, 2018

Place: Room FC1 007, Department of Mathematics, University of Porto

Program

11h00 - Maria Elisa Barbosa Silveira¹: *Drug release enhanced by temperature: an accurate discrete model for solutions in H^3 .*

Abstract: In this talk we consider the coupling between two quasilinear diffusion equations: the diffusion coefficient of the first equation depends on its solution and the diffusion and convective coefficient of the second equation depend on the solution of the first one. This system can be used to describe the drug evolution in a target tissue when the drug transport is enhanced by heat. We study, from an analytical and a numerical viewpoints, the coupling of the heat equation with the drug diffusion equation. A fully discrete piecewise linear finite method is proposed for this system and we establish its stability. Assuming that the heat and the concentration are in H^3 we prove that the introduced method is second order convergent. Numerical experiments illustrating the theoretical results are also included.

11h30 - Nuno Picado²: *Deciding about the emptiness of the interior of a manifold based on a sample of points*

Abstract: In this talk, I will present some results about a method to decide, based on a sample of points from a manifold \mathcal{M} , whether its interior is empty or not. This method was introduced in the article “Stochastic detection of some topological and geometric feature” by Cuevas et al. for independent random variables and sufficiently smooth manifolds. The decision process is based on an estimator constructed using balls centered at the points of the sample. We will show that the approach still holds if the sample satisfies a suitable dependence assumption. After this, the talk will be focused on the construction of models with the type of dependence needed for the results.

12h00 - Dieudonné Mbouna³: *(M, N) -Coherent pairs of order (m, k) and Sobolev Orthogonal Polynomials on the non-uniform lattice*

Abstract: We consider two monic orthogonal polynomial sequences $\{P_n\}_{n \geq 0}$ and $\{Q_n\}_{n \geq 0}$ with respect to regular linear functionals \mathbf{u} and \mathbf{v} , respectively, on a non-uniform lattice $x(s)$. Let $k, m, M, N \in \mathbb{N}_0$ be such

¹Maria Elisa B. Silveira is a PhD student of the Joint PhD Program UC|UP, working at the University of Coimbra, in Numerical Analysis, under the supervision of Professor José Augusto Ferreira.

²Nuno Picado is a PhD student of the Joint PhD Program UC|UP, working at the University of Coimbra, in Probability and Statistics, under the supervision of Professor Paulo Eduardo Oliveira.

³Dieudonné Mbouna is a PhD student of the Joint PhD Program UC|UP, working at the University of Coimbra, in Analysis, under the supervision of Professor José Carlos Petronilho.



that

$$P_n^{[m,x]}(x(s)) + \sum_{j=1}^M a_{j,n} P_{n-j}^{[m,x]}(x(s)) = Q_n^{[k,x]}(x(s)) + \sum_{j=1}^N b_{j,n} Q_{n-j}^{[k,x]}(x(s)), \quad n \in \mathbb{N}_0,$$

where $a_{j,n}$ and $b_{j,n}$ are complex numbers such that $a_{M,n} \neq 0$ if $n \geq M$, $b_{N,n} \neq 0$ if $n \geq 0$, and $a_{j,n} = b_{j,n} = 0$ when $j > n$ and with

$$P_n^{[m,x]}(x(s)) = \frac{\gamma_n!}{\gamma_{m+n}!} D_x^m P_{m+n}(x(s)), \quad n \in \mathbb{N}_0,$$

and the operator D_x is defined on a polynomial P by

$$D_x P(x(s)) = \frac{P(x(s + \frac{1}{2})) - P(x(s - \frac{1}{2}))}{x(s + \frac{1}{2}) - x(s - \frac{1}{2})}.$$

In the first part of the talk, we derive some algebraic properties of the operator D_x and in the second part, we relate this to the well known notion of coherent pairs introduced by Iserles, Koch, Nørsett, and Sanz-Serna. We prove that for an (M, N) - D_x -coherent pair of order (m, k) on a non-uniform lattice $x(s)$, the following results hold. If $m = k$, then the functionals \mathbf{u} and \mathbf{v} are connected by a rational modification, that is $Q(x(s))\mathbf{u} = P(x(s))\mathbf{v}$, for some polynomials P and Q . For $m > k$, \mathbf{u} and $D_x^i S_x^j \mathbf{v}$ for $i + j = m - k$ are semi-classical and they are connected by a rational modification. In the second part of the talk, we connect this notion of coherence to D_x -Sobolev orthogonal polynomials on a non-uniform lattice.